

A DYNAMIC REFERENCE SURFACE FOR HEIGHTS IN CANADA

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The purpose of this paper is twofold, namely (i) to present the developed dynamic geoid model based on the most recent satellite GRACE data and terrestrial data over Canada and (ii) to provide a methodology for incorporating the dynamic geoid as the new height datum in Canada. The move towards a modernized geoid-based vertical datum to replace the existing (and outdated) official height datum CGVD28 involves incorporating the dynamic nature of the regional geoid model, which includes secular changes of up to +1.5 mm/year as a result of the on-going mass transport beneath the uplifting/subsiding Earth's crust. Modelling the regional geoid dynamic variations has been facilitated due to the improved accuracy of the regional geoid model, which is computed using the latest high quality and resolution global gravity models.

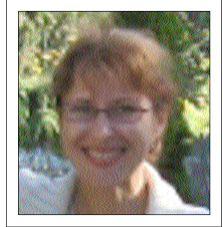
Cet article vise deux buts, notamment : (i) de présenter le modèle dynamique du géoïde basé sur les plus récentes données du satellite GRACE et des données terrestres au-dessus du Canada et (ii) de fournir une méthodologie pour incorporer le géoïde dynamique comme nouvelle référence altimétrique au Canada. Le changement visant à utiliser une nouvelle référence altimétrique modernisée basée sur le géoïde pour remplacer la référence altimétrique officielle (désuète) de 1928 (CGVD28) implique l'incorporation de la nature dynamique du modèle de géoïde régional qui comprend des changements séculaires jusqu'à +1,5 mm/année à cause du transfert continu de masse sous l'écorce terrestre (soulèvement / affaissement). La modélisation des variations dynamiques du géoïde régional a été facilitée grâce à l'exactitude améliorée du modèle de géoïde régional qui est calculé à l'aide des modèles de gravité mondiaux de grande qualité et résolution les plus récents.

1. Introduction

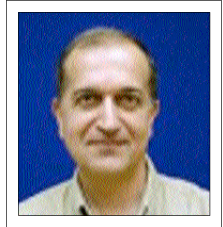
The official reference surface (vertical datum) for orthometric heights in Canada is the Canadian Geodetic Vertical Datum of 1928 (CGVD28), which is constrained to the mean sea level of five tide gauges on the Pacific and Atlantic coasts and is accessible through a vertical control network of more than 80 000 benchmarks distributed over 150 000 km of levelling lines [Véronneau 2002]. Postglacial rebound in this region [Peltier 2004] has caused significant uplift/subsidence of the benchmarks of the vertical control network. Thus, large systematic errors exist in the network, which adds to the significant east-west tilting because of the datum's mean sea level constraints. Moreover, local motion of the benchmarks, limited spatial coverage and accessibility, and the high maintenance cost of the control network have led towards the need for a modernized vertical datum based on the most accurate regional geoid model [Véronneau et al. 2006]. This new datum will be accessible via Global Navigation Satellite System (GNSS) positioning. It is also envisioned that the new Canadian geoid-based datum will be compatible with a future global vertical datum (world height system), which is

crucial for studies related to large-scale geodynamics and geo-hazards processes.

The accurate determination of orthometric heights via GNSS/levelling requires a centimetre(s) accuracy of the regional geoid [Huang et al. 2006]. The present-day theory for computing the regional geoid for Canada meets this requirement [Vaníček et al. 1999]. From a practical viewpoint, however, several important factors need to be addressed, namely (i) incorporating the latest and most accurate global geoid models provided by Gravity Recovery and Climate Experiment (GRACE) and the recently-launched Gravity Field and Steady-state Ocean Circulation Explorer (GOCE) satellite missions, (ii) updating the regional gravity database, (iii) using high accuracy digital terrain and crustal density models, and (iv) incorporating the dynamic changes in the geoid in the new vertical datum. The geoid in North America experiences large secular rise of up to 1.5 mm/year (in the vicinity of Hudson Bay) as a result of the on-going mass transport beneath the uplifting/subsiding Earth's crust as well as secular changes in sea level. Over a time period of 5 to 10



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years, the magnitude of the secular geoid changes becomes as large as the desirable “one-centimetre” accuracy of the geoid.

Various local empirical postglacial rebound models have been developed in Canada and North America using solely geodetic observations: GPS velocities [Henton *et al.* 2006; Sella *et al.* 2007], re-levelled height differences and tide gauge sea level velocities [Mainville and Craymer 2005], repeat absolute and relative gravity measurements [Pagiatakis and Salib 2003], joint tide gauge and satellite altimetry data [Braun *et al.* 2008]. The GRACE mission, however, has provided (for the first time) estimates of the spatially homogeneous temporal variations of the geoid on a regional and global scale [Tapley *et al.* 2004], allowing comprehensive studies of postglacial rebound through a combination with the local geodetic measurements [van der Wal *et al.* 2008]. Figure 1 shows the geoid secular trend computed over a time interval of 6 years, from the GRACE data distributed by the Center for Space Research (CSR), University of Texas.

In this paper, the estimation of the temporal changes of the regional geoid via a combination of GRACE, local GPS, and gravity data is presented. Issues related to the continental hydrology signals interfering with the postglacial rebound signal at the short time scales of the GRACE observations are also addressed. Finally, a methodology for implementing the dynamic geoid-based datum for heights in Canada is developed.

2. Modelling the Dynamic Geoid with GRACE and Terrestrial Data

The dynamic geoid height can be represented as a function of the location (φ, λ) and time epoch t as

$$N(\varphi, \lambda, t) = N^{\text{stat}}(\varphi, \lambda) + \dot{N}(\varphi, \lambda)t + \tilde{N}(\varphi, \lambda, t) \quad (1)$$

where $N^{\text{stat}}(\varphi, \lambda)$ is the static geoid height and $\dot{N}(\varphi, \lambda)$ is the trend (rate of change) of the geoid. The last term $\tilde{N}(\varphi, \lambda, t)$ represents semi-annual, annual, non-periodic, and episodic changes in the geoid height over the time period of interest. The main periodic signal in the geoid height in North America is the annual cycle of snow mass accumulation and melting. Other temporal variations of interest are the decadal signal of mass loss in the Alaskan glaciers, Greenland ice sheet and the trend-like changes in the continental hydrology over the analyzed time period [Range Iova *et al.* 2007].

For the purpose of this study, we simplify eq.(1) to account only for the trend term as follows:

$$N(\varphi, \lambda, t) = N(\varphi, \lambda, t_0) + \dot{N}(\varphi, \lambda)(t - t_0) \quad (2)$$

In this equation, $N(\varphi, \lambda, t_0)$ is the geoid model for the reference epoch t_0 . The geoid height for any arbitrary epoch t is obtained by a simple linear

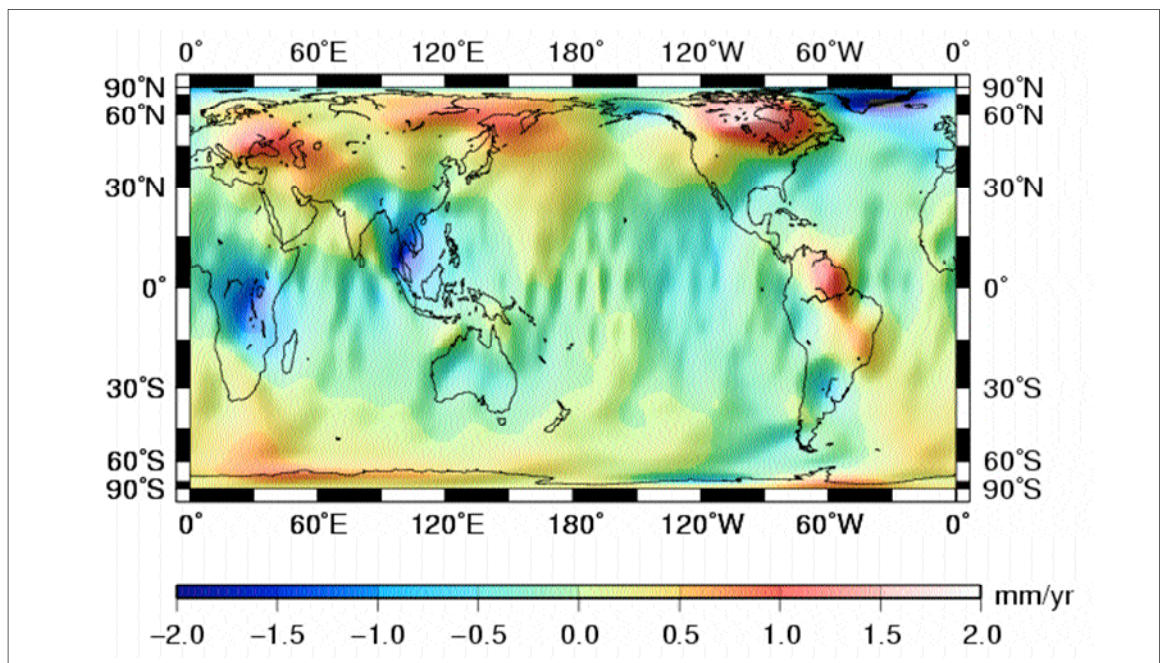


Figure 1: GRACE-observed geoid rates derived from the Center for Space Research (CSR) data.

interpolation. All non-linear terms in eq.(1) are regarded as noise that may obscure the estimation of the geoid trend when relatively short GRACE time series are analyzed.

2.1. GRACE-Derived Dynamic Geoid

GRACE observes changes in the geoid that are integrated in a vertical column and spatially averaged at distances of hundreds to thousands of kilometres. These changes include large-scale (global and regional) annual, semi-annual, trend, and episodic geophysical signals in the Earth's interior, on land, at the ocean, and in the atmosphere. Separation of signals from different sources is impossible without a priori available models of the different mass variations. As shown by *Rangelova et al.* [2007], in the previously glaciated areas in North America, the postglacial rebound signal can interfere with the hydrology signals. For example, one of the largest snow-related signals is located in the Quebec-Labrador region, where the maximum geoid secular changes are also observed. Moreover, the intra-annual hydrology variations may appear trend-like over the (now) seven-year duration of the GRACE observations. To estimate the secular geoid rate, the hydrology signal from the available continental water storage models is removed a priori, though the improvement of the same hydrology models relies on the knowledge gained from GRACE to constrain the postglacial rebound models. Other signals of interest in North America are the present-day melting of the Greenland ice sheet and Alaska glaciers. According to *van der Wal et al.* [2009], the leakage of Greenland and Alaska signals is between -0.4 and -0.2 mm/yr for the northern areas of Hudson Bay, depending on the degree of smoothing. Another geophysical signal is the sea level rise due to the accelerated melting of polar ice sheets and mountain glaciers, which in Hudson Bay has a similar to postglacial rebound pattern but smaller magnitude [*Wahr and Davis* 2002].

We present the geoid rates computed with hydrology corrections of the two most advanced global continental water storage models, namely the Global Land Data Assimilation System (GLDAS) of *Rodell et al.* [2004] and the WaterGAP Global Hydrology Model (WGHM) of *Döll et al.* [2003] and *Hunger and Döll* [2008]. The former model accounts for water storage in soil, snow, canopy, and groundwater storage while the latter model also includes water stored in rivers, lakes, and reservoirs as well as anthropogenic water use. For the models in Figures 2a and b, two peaks are observed west and southeast of Hudson Bay. The GRACE/GLDAS geoid rate peaks at 1.4 ± 0.05

mm/yr at both locations. The GRACE/WGHM geoid rate has a larger peak of 1.5 ± 0.05 mm/yr southeast of Hudson Bay. The geoid rate error is computed using the calibrated standard deviations provided by the GRACE project. Note that the difference in the geoid rate peak is above the error, but it can be directly interpreted as an uncertainty of the water storage models. The peak locations account for the multi-dome structure of the North American ice sheet complex, but postglacial rebound predictions using the ICE-5G de-glaciation model show the larger peak signal west of Hudson Bay [*Peltier* 2004]. Evidently, the derived geoid rate in North America depends on the hydrology correction applied, and generally is larger when the WGHM model is used. Figure 2c shows the difference of the two models. While there is a smaller change in the pattern in the proximity of Hudson Bay, the models vary significantly over the western areas, showing the uncertainties of the snow-modelling in the Rockies and southeastern Alaska glaciers area.

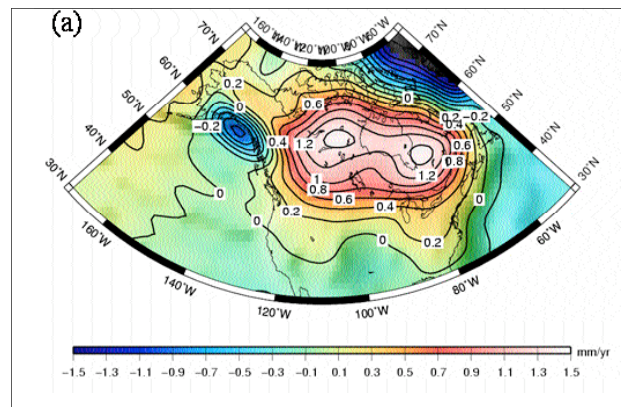


Figure 2a: The GRACE-derived geoid rate with the GLDAS/Noah hydrology corrections.

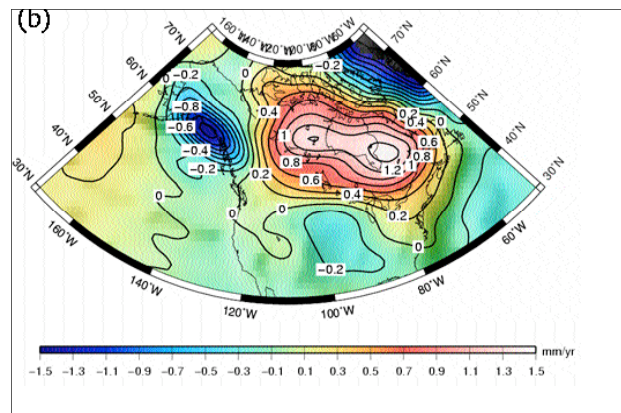


Figure 2b: The GRACE-derived geoid rate with the WGHM hydrology corrections.

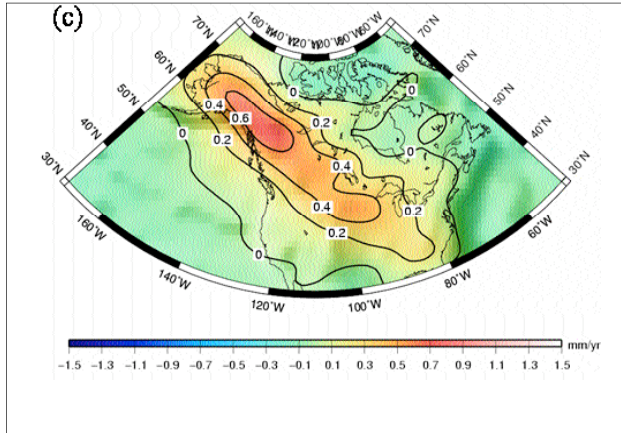


Figure 2c: The GRACE-derived geoid rate differences of the two models.

2.2. Dynamic Geoid from Combined GRACE, GPS, and Gravity Data

To verify postglacial rebound simulation outputs, the geodynamics modelling relies heavily on the available geodetic (gravimetric and geometric) constraints that sample the largest (in magnitude and spatial scale) secular signal in North America. Collocated permanent GNSS stations and absolute gravity sites provide complementary information in terms of rates of absolute vertical displacement (geometric component) and change of terrestrial gravity (gravimetric component) that is useful for validation of the GRACE-observed rates.

Figure 3 shows the primary Canadian Gravity Standardization Network (CGSN) (red markers), which was designed to provide gravimetric control

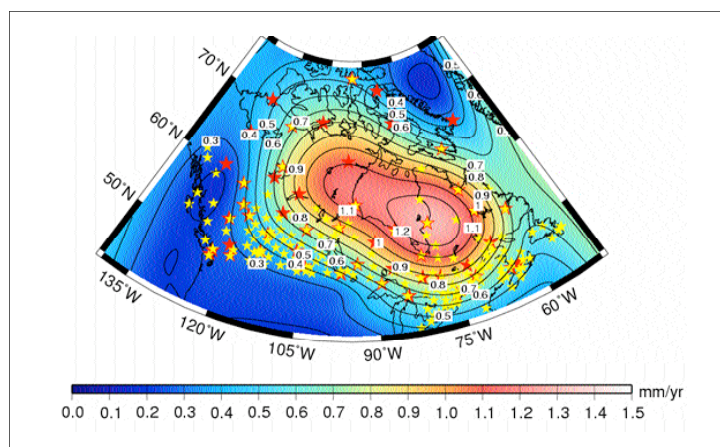


Figure 3: Combined geoid rates from GRACE, absolute gravity (red markers), and GPS velocities (yellow markers).

and to support the geophysical exploration in Canada, but it was not intended for geodynamics monitoring [Pagiatakis and Salib 2003]. The CGSN consists of about 1 500 primary and secondary gravity reference stations; 64 of them are classified as “primary gravity control” sites. Five absolute gravity stations were re-observed at least three times. The epoch of the latest adjustment of the network is 2000.0, and the provided absolute gravity rates refer to the same epoch. The CGSN sites are sparse but effectively sample the characteristic wavelengths of the postglacial rebound signal.

The Canadian Base Network (CBN), with yellow markers in Figure 3, provides velocity estimates by combining longer than a decade permanent and episodic GPS measurements [Henton *et al.* 2006]. The GPS vertical velocities have a spatial pattern and magnitude consistent with postglacial rebound model predictions. The GPS stations are clustered in southern Canada with few exceptions to the north of the degree 60 parallel. The CGSN sites and the CBN stations are not collocated and the measurements of the two networks have different time spans. Currently, a modernization of CGSN is in process with absolute gravity observations being collocated with GNSS measurements, thus providing constraints for postglacial rebound modelling and a basis for the new geoid-based height datum in Canada [Henton *et al.* 2005].

The geoid rates in Figure 3 are obtained by means of a combined procedure of least-squares collocation, outlier detection, and variance component estimation. This procedure is described in detail in Rangelova and Sideris [2008]. Here, we focus on the possible mitigation of some errors pertinent to the GRACE-derived rates by making use of the terrestrial data. A noticeable difference between the geoid rates in Figures 2 and 3 is the reduction of the magnitude of the peak secular signal. This is mostly due to proper calibration of the GRACE errors, which results in filtering out part of the large peak signal. Another important advantage of the combined model is the reduction of the leakage of the outside signals over the area of interest as well as the mitigation of the uncertainties of the hydrology corrections. For example, though sparse, the gravity rates available in the northeast areas help to reduce the large negative leakage signal from the melting of the Greenland ice sheet. On the other hand, the obscurity of the terrestrial data west of Hudson Bay precludes a definite conclusion about the location and magnitude of the large peak seen by GRACE. This again demonstrates the importance of more and densely distributed terrestrial measurements in the areas with the largest secular signal.

3. Dynamic Height Reference Surface

The effect of the secular dynamic variations on the geoid-based datum is studied by means of a combined least-squares adjustment analysis of ellipsoidal, orthometric and geoid heights, where the observation equation is as follows:

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{v}, \mathbf{C}_v \quad (3)$$

where $\mathbf{l} = [l_1 l_2 \dots l_n]^T$ is a vector of n height misclosures $l = h - N - H$ at the benchmarks of the vertical control network computed with the ellipsoidal (GPS) height, h , geoid height, N , and orthometric height, H . The vector $\mathbf{x} = [x_1 x_2 \dots x_k]^T$ contains k unknown parameters; \mathbf{A} is the coefficient matrix with vector rows $\mathbf{a}_i = [a_1 a_2 \dots a_k]$, $i = 1, \dots, n$; and $\mathbf{v} = [v_1 v_2 \dots v_n]^T$ is a vector of residuals. The covariance matrix \mathbf{C}_v is defined as $\mathbf{C}_v = \mathbf{C}_h + \mathbf{C}_H + \mathbf{C}_N$ with, \mathbf{C}_h , \mathbf{C}_H and \mathbf{C}_N being the fully-populated error covariance matrices for the ellipsoidal, orthometric, and geoid heights, respectively. The parametric component in eq.(3), which accounts for the systematic errors among the ellipsoidal, orthometric, and geoid heights, is represented as follows [e.g. Fotopoulos 2005]:

$$\mathbf{a}_i^T \mathbf{x} = x_1 + x_2 \cos \varphi_i \cos \lambda_i + x_3 \cos \varphi_i \sin \lambda_i + x_4 \sin \varphi_i \quad (4)$$

where (φ_i, λ_i) is the geographical location of an arbitrary point i . The error of the parametric component at this location is $\hat{\sigma}_i^a = \sqrt{\mathbf{a}_i^T \mathbf{C}_\hat{\mathbf{x}} \mathbf{a}_i}$, where

$$\mathbf{C}_\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{C}_v^{-1} \mathbf{A})^{-1}.$$

To study the effect of the secular dynamic changes in the geoid and vertical crustal displacement, we assume that the geoid height $N(\varphi, \lambda, t_0)$ is computed from a geoid model for the epoch t_0 . The orthometric height at epoch t is computed from the measured ellipsoidal height $h(\varphi, \lambda, t)$ and the geoid height as follows:

$$H(\varphi, \lambda, t) = h(\varphi, \lambda, t) - N(\varphi, \lambda, t_0) - (\mathbf{a}^T \hat{\mathbf{x}})_t \quad (5)$$

where the parametric component computed with the estimated vector of parameters $\hat{\mathbf{x}}$

$$(\mathbf{a}^T \hat{\mathbf{x}})_t = (\mathbf{a}^T \hat{\mathbf{x}})_0 + \delta N + \delta H \quad (6)$$

is a sum of the main component $(\mathbf{a}^T \hat{\mathbf{x}})_0$ at epoch t_0 and the temporal changes in the geoid height, $\delta N = (t - t_0)\dot{N}$, and orthometric height, $\delta H = (t - t_0)\dot{H}$,

for the time interval $(t - t_0)$. The fundamental point of the vertical datum is assumed to be unaffected by the secular vertical crustal motion. Also, the equipotential surface through the fundamental point does not change with time.

We assess the significance of the dynamic geoid and crustal motion on a decadal time scale via a comparison with calibrated errors of the ellipsoidal, geoid, and orthometric heights. Two relative

variables, $\max((\delta N)_i / \hat{\sigma}_i^a)$ and $\max((\delta N + \delta H)_i / \hat{\sigma}_i^a)$ are defined with the maximum ratios of the temporal change and the error of the parametric component for all of the points in the area of interest. Values larger than 1 indicate that the secular effect is not negligible compared to the accuracy with which the systematic errors of the three height components are estimated. Furthermore, we also use the maximum ratio of the mean standard deviation $\bar{\sigma}_\vartheta$ for each of the three height components and the error of the parametric component, i.e. $\max(\bar{\sigma}_\vartheta / \hat{\sigma}_i^a)$, $\vartheta = h, H, N$

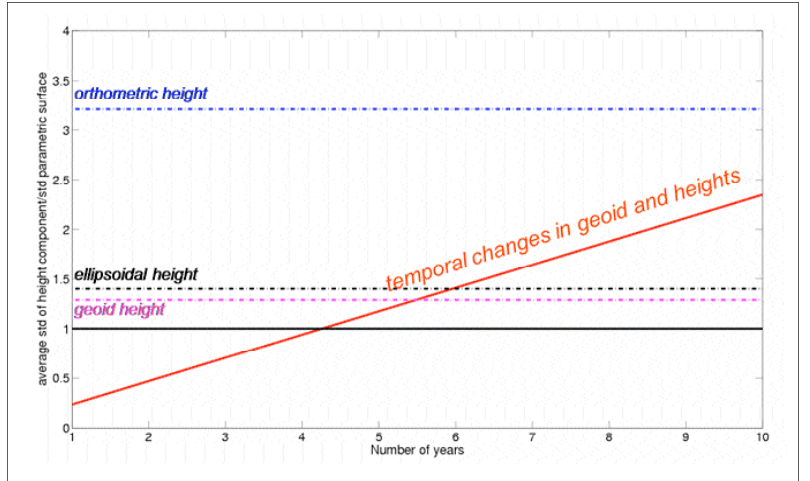


Figure 4: Accumulated effect of the secular change in the geoid and crustal motion (relative to the geoid) compared to the errors of the ellipsoidal, orthometric, and geoid heights.

We use a set of 430 GPS-on-benchmark points in Canada (Geodetic Survey Division, Natural Resources, Canada), which is used in the error calibration of the CGG2005 geoid model [Huang *et al.* 2006]. The ellipsoidal heights are determined from the campaign GPS surveys after 1994 in the ITRF1997 reference frame. The orthometric heights are computed via a minimally constrained least-squares adjustment of the levelling measurements after 1981 with a single reference datum point in Rimouski (Quebec), and the vertical crustal motion is accounted for in the least-squares adjustment procedure [Véronneau 2002]. All point

height misclosures are made available together with the calibrated fully-populated covariance matrices of the ellipsoidal, orthometric, and geoid heights. We interpolate the geoid rates and the relative (to the geoid) vertical crustal motion, which is also the rate of change of orthometric height, at the GPS-on-benchmarks points using the combined GRACE and terrestrial data models (section 2.2).

The accumulated effect of the dynamic changes for the time elapsed since the epoch t_0 is shown in Figure 4. It exceeds the error of the parametric component after 4 years and the errors of the geoid and ellipsoidal heights after 5 to 6 years. However, the dynamic effect is still below the mean error of the orthometric heights. If the geoid change is taken into account solely in eq.(6), the dynamic effect remains far below the current errors of the three height components. Therefore, the contemporary errors of the three height components obscure the incorporation of the dynamic geoid in the height datum. On the other hand, the effect of the vertical crustal motion is significant and should be taken into account in the GNSS/levelling.

Furthermore, the significance of the dynamic components depends on the proper relative weighting of the ellipsoidal, orthometric, and geoid heights. The dynamic effect is overestimated if diagonal instead of fully-populated covariance matrices are used. This may lead to the incorrect conclusion that the vertical reference surface should be corrected for the temporal effect more frequently. A further study indicates that if the errors of the three height components decrease to 10-15 mm, the geoid heights should be corrected for the dynamic effect every 8 to 10 years, and the vertical crustal motion should be accounted for every 2 years.

In lieu of the goal of achieving a “one centimetre” accuracy of the regional geoid, the following recommendations can be made for establishing a dynamic geoid-based vertical datum in Canada:

a. *Definition of the datum.* The height reference surface should be defined by a regional geoid model computed for one reference epoch with all terrestrial data referring to the same epoch. The ellipsoidal and orthometric heights should be corrected for the vertical crustal displacement and referenced to the epoch of the geoid model. Calibrated errors of the geoid model should be computed by means of a combined least-squares adjustment of ellipsoidal, orthometric, and geoid heights. Even a geoid-based vertical datum will require some GPS-on-benchmarks points distributed in the region in order to periodically conduct validation and calibration studies. To obtain temporally homogeneous height

data, consistent models of the rates of change of the geoid and vertical crustal displacement should be used. We recommend that combined GRACE and terrestrial data models (such as the model in Figure 3) are used due to (i) the reduced effect of the errors in the GRACE-derived rates and (ii) the observed better agreement with outputs from postglacial rebound model simulations.

b. *Assessment of the datum stability.* A criterion for *stability* of the vertical reference surface should be introduced in terms of the mean calibrated error of the geoid model. The vertical reference surface should be assumed *stable* if the secular dynamic changes in the geoid height for the time elapsed from the reference epoch remain below the mean calibrated geoid error. The vertical reference surface should be corrected if the stability requirement is no longer met.

4. Conclusions

A study on modelling the effect of secular changes in the geoid and vertical crustal motion on the reference surface for orthometric heights in Canada was presented. A large number of science and engineering users (for example in oceanography, hydrology, GIS, geo-hazard monitoring, geodynamics, to name a few) are in need of precisely defined vertical reference surface (datum) that also includes temporal variations.

The models developed in our study make use of the GRACE-observed mass variations delivered by the satellite mission with a homogenous global coverage and high accuracy for the last seven years. The secular postglacial rebound signal can be recovered from the GRACE observations upon accurate knowledge of the continental hydrology mass variations, which is expected to be improved by the same GRACE observations. While the uncertainties in the continental hydrology models are significant, they will be of less importance in the future with the increase of the time span of the GRACE mission. Moreover, since the errors of the terrestrial data (GPS and absolute gravity) are different in nature from the GRACE errors, the combined use of both ultimately offers advantages mainly seen in the mitigation of the inevitable leakage effect in the GRACE-observed geoid changes.

Finally, we demonstrated that the present day errors of the geoid and heights in Canada are larger than the secular geoid changes on a decadal scale. Ultimately, the dynamic geoid change will not be negligible when the geoid error reaches the

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so-called “one centimetre” level. In fact, the use of dynamic geoid heights depends on the accuracy requirements by different users. For example, sea level studies require a dynamic geoid with a millimetre level of accuracy, while positioning and navigation may not be interested in the temporal change of the height reference surface on shorter than a decade time intervals. Within this time interval, however, for accurate scientific applications, the geoid height can be corrected by means of models such as those developed in this study.

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