DEVELOPING AN ADAPTIVE TOPOLOGICAL TESSELLATION FOR 3D MODELING IN GEOSCIENCES

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The value of Geographic Information Systems (GIS) is widely known in a variety of geoscientific applications ranging from water resources management to the study of global warming impacts. GIS can provide geoscientists with strong computing platforms for spatial data management, visualization, querying, integration, and analysis. However, representation and management of geoscientific phenomena which are usually 3D and heterogeneous mostly require a 3D optimal spatial tessellation. An optimal tessellation is characterized by well-shaped and well-spaced elements that provide an accurate representation of topological and geometrical information. In this paper, we discuss the limitations of current three-dimensional spatial tessellation methods that are used in 3D geological modeling, 3D interpolation, and 3D fluid flow simulations, etc. Then, we propose an automatic refinement algorithm based on Delaunay tetrahedralisation and its dynamic operations. Finally, we present and discuss the results and the performance of the algorithm.

La valeur des systèmes d'information géographique (SIG) est grandement connue pour un ensemble d'applications géoscientifiques variant de la gestion des ressources en eau à l'étude des répercussions du réchauffement planétaire. Les SIG fournissent aux géoscientifiques des plates-formes de calcul rigoureuses pour la gestion, la visualisation, les requêtes, l'intégration et l'analyse des données spatiales. Toutefois, la représentation et la gestion des phénomènes géoscientifiques qui sont généralement tridimensionnels et hétérogènes exigent une tessellation spatiale optimale en 3-D. Une tessellation optimale est caractérisée par des éléments bien modélisés et bien espacés qui fournissent une représentation exacte de l'information topologique et géométrique. Dans le présent article, nous discutons des limites des méthodes de tessellation spatiale tridimensionnelle actuelles qui sont utilisées pour la modélisation géologique 3-D, l'interpolation 3-D, les simulations de l'écoulement des fluides 3-D, entre autres. Ensuite, nous proposons un algorithme de raffinement automatique basé sur la tétraédralisation de Delaunay et ses opérations dynamiques. Enfin, nous présentons et discutons des résultats et du rendement de l'algorithme.

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1. Introduction

Geographic Information Systems (GIS) are widely used for modeling, representation, management, and analysis of spatial data in many disciplines. In particular, geoscientists increasingly use these tools in many environmental applications ranging from water resources management to the study of global warming impacts. *Raines* [2007] presents a very interesting review on the variety of applications of GIS-based spatial modeling to the problems in geosciences. In most environmental applications, geoscientists need to deal with the third spatial dimension z. For example, in geological modeling, structures such as faults cannot be represented by 2D or even 2.5D modeling tools where for each horizontal location several z values exist. Therefore, 3D modeling is more appropriate to deal with these kinds of phenomena.

Measurement techniques in geosciences, such as seismic profiling, magnetic or gravity surveys and borehole drilling in geology, and the uses of weather balloons and probes in metrology, often result in points in 3D space x,y,z with irregular distribution. Each data point is defined by its location in 3D space, and can have one or more attributes attached to it. To represent a spatial object from a set of discrete samples, it is necessary to tessellate the objects into finite elements such that the union of all elements completely fills the object [*Raper* 2000; *Worboys and Duckhum* 2004]. To each element of the tessellation, a set of attributes such as mass, porosity, etc., can be assigned in order to characterize the local nature of the object.

In some geoscientific applications such as geology, the shape and distribution of the objects can be highly irregular and complex. Therefore, the tessellation elements must not only be well-shaped, but also should represent the complexity of the objects as accurately as possible. In fact, the tessellation elements are used to form an approximation function based on physical variables (permeability, porosity, etc.), and their sizes and shapes thus affect the accuracy of the approximation as well as the computational costs [*Mansell et al.* 2002]. As an example, specific regions with high variation of the properties require a tessellation with fine resolution. However, because the time required for modeling of the objects is proportional to the number of tessellation elements, the creation of a uniformly fine tessellation is computationally costly. Consequently, the determination of optimal resolution and shape of the elements of a tessellation are critical issues that remain among the most challenging problems in 3D spatial modeling in geosciences [*Bower et al*. 2005; *Lepage* 2003; *Shewchuk* 2002].

Many methods have been already proposed for an automatic tessellation and refinement for different purposes, including interpolation, rendering, numerical modeling, and simulation applications [*Shewchuk* 2002]. These methods have been mostly developed in other disciplines such as in computational geometry, which are generally based on theoretical considerations and do not consider the physical nature of the problems in geosciences [*Cordes and Putti* 2001]. In order to overcome the limitations of the existing methods, this paper proposes and implements a new method for automatic generation and refinement of an adaptive tessellation for geoscientific applications, based on Delaunay tetrahedralisation and 3D Voronoi diagram, that takes into account the geometrical, topological, and physical properties of the phenomena to be modeled. The proposed method provides the possibility of dynamic and local edition and manipulation of the tessellation, and allows users an on-the-fly interaction with the 3D model which is not possible with other existing static mesh generation methods.

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The remainder of this paper is organized as follows. Section 2 explains the current approaches and their limitations for 3D optimal tessellation generation. Section 3 discusses 3D Delaunay-based tessellations. Section 4 explains the proposed solution to these limitations through the development of an adaptive topological tessellation algorithm for 3D modeling in geosciences. In section 5, the proposed algorithm is applied to a set of data for evaluation purposes. Finally, the conclusion of the paper discusses the efficiency of the proposed algorithm.

2. Literature Review

The tessellations used for 3D modeling in geoscientific applications are either regular or irregular, also referred to as structured or unstructured, tessellations [*Lepage* 2003]. The elements of a regular tessellation have uniform shapes and sizes, such as 2D rectangles and pixels or 3D cubes and voxels. For example, raster-based digital elevation models (DEM), rainfall, and temperature maps use 2D regular tessellations, whereas Rock-property modeling uses 3D tessellations [*Bosch* 1998; *Jessell* 2001]. In a regular spatial tessellation, the adjacency relations between elements of the tessellation are implicit and can be readily obtained. However, a major limitation of a regular tessellation is related to its uniform resolution. Representation of a phenomenon with a high spatial variability requires a grid with a very fine resolution. Working with such a grid, especially in a 3D environment, is computationally very costly. To tackle this problem, hierarchical tessellations such as Quadtree in 2D and Octree in 3D can be used [*Samet* 1984]. These methods subdivide the space into four squares (Quadtree, in 2D) or eight cubes (Octree, in 3D) of equal sizes until either each element contains a homogeneous region or reaches a desired resolution [*Li* 1996]. Therefore, in these hierarchical models, a coarse resolution is used to represent large homogeneous areas, while a finer resolution is used for areas of high spatial variability. Although these methods reduce the amount of required memory storage, a small change in data may result in a quite different tessellation in the case of the representation of a dynamic phenomenon [*de Berg et al.* 2000; *Worboys and Duckham* 2004].

The elements of an irregular tessellation can posess any size and shape (for example, triangles and polygons in 2D, or tetrahedrons and polyhedrons in 3D) and can thus fill any arbitrary surface or volume. Contrary to regular tessellations, the topology needs to be computed and stored explicitly, which is relatively complex for 3D irregular meshes. Despite the complexity of the computation of 3D topological relationships, irregular tessellations are increasingly popular [*Soni* 2000] because they are very flexible and can represent complex geometries using very irregularly distributed data. GOCAD,[1] LaGgriT,[2] GEOMESH [*Gable et al.* 1995], and HydroGeoSphere [*Therrien et al.* 2007] are examples of softwares that generate irregular tessellations for geoscientific applications. In HydroGeoSphere, for example, the space is discretized to triangular prisms which are created from rectangular prisms (Figure 1). Although this type of tessellation has a simple structure, it is less flexible for representing all the complexities of the phenomena that can be found in geoscientific applications.

[1] *http://www.gocad.org/www/*

[2] *http://lagrit.lanl.gov/*

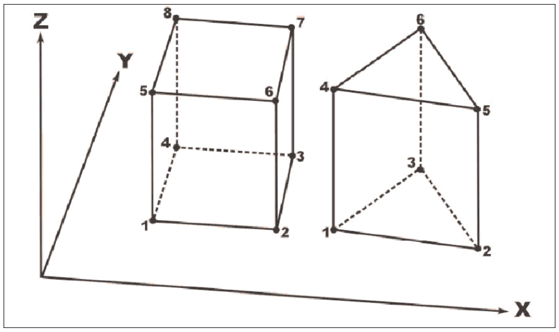


Figure 1: Simple regular and irregular elements in HydroGeoSphere [*Therrien et al.* 2007].

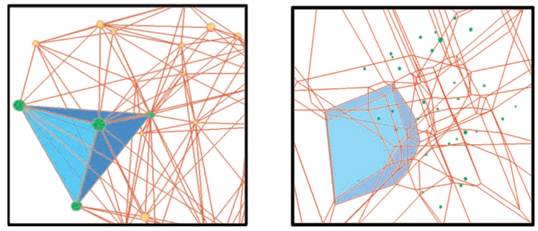
Delaunay tetrahedralization (DT) and 3D Voronoi diagram (VD) provide an adequate tessellation for modeling and representation purposes in 2D and 3D spaces [*Aurenhammer* 1987; *Raja* 1991; *Shewchuk* 1997; *Okabe* 2001; *Mostafavi* 2002; *Ledoux and Gold* 2008; *Hashemi Beni et al*. 2008; *Shewchuk* 2008]. In 3D space, Delaunay tetrahedra and 3D Voronoi cells can be defined by points with an arbitrary distribution, creating elements of different sizes and shapes which can fill any arbitrary surface or volume. In addition, each 3D Voronoi cell can have an arbitrary number of neighbors with clearly defined adjacency relations that can be explicitly retrieved if needed. In the next section, the application of DT and VD for 3D modeling of geoscientific phenomena, and their potential and limitations, are discussed.

3. Delaunay- and Voronoi-based Tessellation in Geosciences

3.1 Definitions

A Delaunay tetrahedralization (DT) for a set of data points is constructed by partitioning the 3D space into tetrahedra based on the empty circumsphere criterion. The criterion ensures that the circumsphere of each tetrahedron does not contain any other point of the set (Figure 2). The resulting tessellation is unique for the point set, except when degenerated cases in the data set exist (if five or more points are co-spherical in 3D).

Voronoi diagram (VD) is the dual graph of DT, which can be very useful for geoscientific applications. The VD for a set of points in a 3D Euclidean space is constructed by partitioning of the space in such a way that each location in the space is assigned to the point of the set closest to the location with respect to the Euclidean distance.



(a) **(b)**

Figure 2: (a) Delaunay tetrahedralisation, (b) 3D Voronoi diagram.

The duality between VD and DT is based on specific correspondences between geometric elements of the two data structures. In a 3D space, each Delaunay tetrahedron corresponds to a Voronoi vertex, each Delaunay triangular face becomes a Voronoi edge, a Delaunay edge corresponds to a Voronoi face, and finally, each Delaunay vertex corresponds to a Voronoi polyhedron and vice versa. This property allows extraction of the Voronoi tessellation from the Delaunay tessellation and vice versa.

3.2 Application of 3D DT and VD for 3D Geoscientific Modeling and Its Limitations

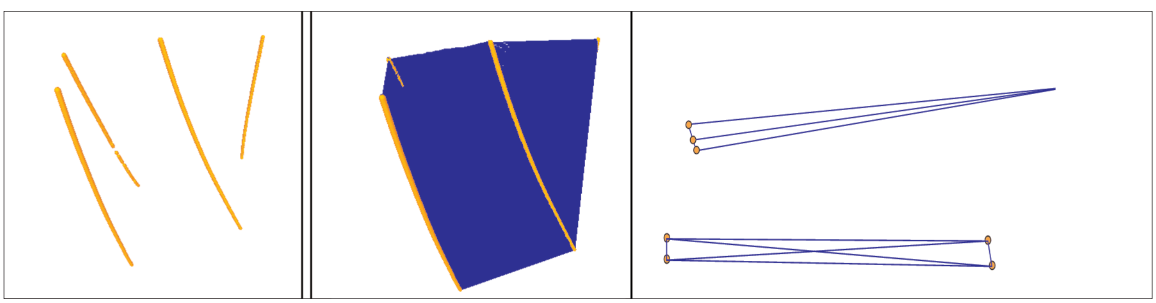
Delaunay tetrahedralisation and 3D VD are already applied to 3D spatial modeling purposes in geosciences. *Courrioux et al.* [2001] demonstrate that volume reconstruction using the 3D VD is suitable for geological objects, since it gives a consistent partition of space according to the data specification. *Hale* [2002] applied DT and VD to reservoir simulation using 3D seismic images, and demonstrated the potentials of the geometric structures for fluid flow simulation during all the steps of the seismic interpretation, fault framework building, and the reservoir modeling. *Carette et al*. [2008] have applied 3D DT and VD for the modeling of marine environment. They used a DT integrated with a clustering algorithm for the representation and analysis of fish aggregations using acoustic data.

Despite these advantages, the use of the VD and DT to model 3D geoscientific datasets which are not randomly distributed is very challenging [*Lattuada and Raper* 1995; *Raper* 2000]. For example, for the simulation of ground water flow in a hydrogeological system, data may be sampled either directly from prospective drills, well logs, or boreholes, or indirectly from seismic, magnetic, or gravity surveys. These data form a highly irregular set of distributed points, sometimes aligned very densely on the *z* axis. It is from those data sets that we create a model which represents, for example, a complex hydrogeological system composed of geological objects such as fractures, faults, and layers. A 3D Voronoi tessellation of such a data set does not properly represent the discontinuities and boundaries within the complex system. This is because in a VD, a volume of influence is assigned to each point where physical properties are assumed to be constant. To overcome this limitation, some efforts [*Lattuada and Raper* 1995; *Schaap and deWeygaert* 2000] have been made to use DT instead of using VD; however, the DT has its own limitations for representing the physical discontinuities. The elements of the DT may not satisfy shape and size criteria for a specific application [*Shewchuk* 1997]. For example, the tetrahedra may be smaller or larger than a desired size. This means that some points are not "well-spaced." Some angles within tetrahedra may be too small or too large, and hence they are not "well-shaped" (Figure 3).

As previously mentioned, tessellation elements are frequently used in geoscientific applications to form a piecewise linear approximation of a function based on physical variables (permeability, porosity), and their sizes and shapes affect the accuracy of the approximation [*Shewchuk* 1997]. Hence, poorly-shaped elements are not adequate for such an approximation, and that is why a refinement process is usually applied following the initial tessellation construction. In fact, the tessellation refinement aims to obtain good quality (well-shaped and well-spaced) tessellation elements for a given application, which is discussed in following sections.

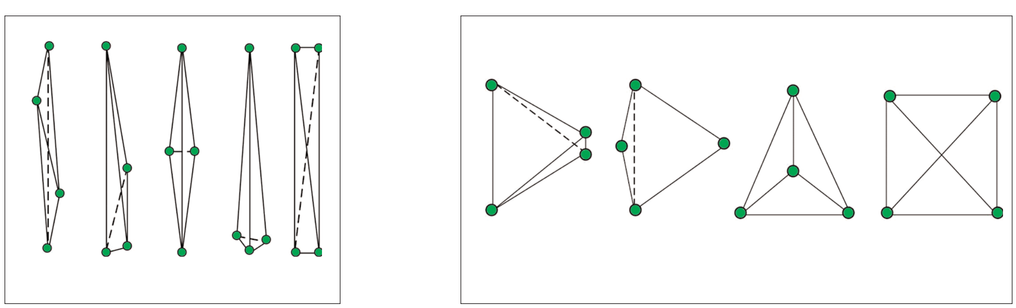
3.3 Limitations of the DT for Geoscientific Modeling

Depending on the distribution of the data used for the creation of a DT, there may be several types of poor-quality tetrahedra in the tessellation. As *Shewchuk* [1997] states, poor quality tetrahedra are typically flat or skinny. In a skinny tetrahedron, the vertices are close to a line, and in a flat tetrahedron, one of the vertices is close to the plane formed by the other three vertices. A skinny tetrahedron has a circumradius much larger than its shortest edge. Therefore, the circumradius-to-shortest-edge ratio of a tetrahedron, or the ratio of the circumradius to the inscribed sphere radius, is the most important criterion for detecting a skinny tetrahedron [*Si* 2006]. These tetrahedra can appear as one of five types, as illustrated in Figure 4a [*Edelsbrunner* 2001]. Flat tetrahedra (Figure 4b) can have a small circumradius-to-shortest-edge ratio. However, their volume is very small (approaching zero) because one of their vertices is close to the plane formed by the other three vertices of the tetrahedron. A tetrahedron with



(a) (b) (c)

Figure 3: (a) A data set from four boreholes, which are abundant vertically but very sparse horizontally; (b) Delaunay tessellation of the data set; (c) Examples of two elements with poor quality in the tessellation.



(a) (b)

Figure 4: (a) Five types of skinny tetrahedra [*Edelsbrunner* 2001); (b) Four types of flat tetrahedra; the rightmost tetrahedron is a sliver.

almost zero volume is called a sliver. A combination of measures, such as the ratios of maximum edge length to inscribed sphere radius, and the ratio of the maximum edge length to the minimum edge length, can be used to distinguish a skinny tetrahedron from a flat tetrahedron [*Parthasarathy et al.* 1993]. In some methods, the relation between the volume and the face area or edge length is important. For example, *de Cougny et al*. [1999] define a normalized aspect ratio based on the tetrahedron volume and its four facet areas. *Dannenlogue and Tanguy* [1991] use a ratio involving the tetrahedron volume and the average edge length of the tetrahedron edges to determine a flat tetrahedron.

As mentioned, all the criteria to identify poor-quality tetrahedra are purely geometrical and do not consider the physical nature of the system. For example, a tetrahedron meeting all these geometric constraints will be poor quality for the simulation purpose if its size is larger than the given resolution. However, it will not be problematic if a tetrahedron, located in the given area with a very smooth variation of physical values, does not perfectly meet the mentioned criteria. In addition, in using these geometrical criteria, the refinement for tessellation conforming to geoscientific data can be very time-consuming, due to the special configuration of the sampled points on one hand, and geometry complexities of the geoscientific objects and phenomena on the other hand.

Poor-quality tetrahedra handling: Poor-quality tetrahedra are unsuitable for 3D modeling of geosciences objects and phenomena and must be modified or removed from the tessellation. In order to treat the poor-quality tetrahedra, the new points are carefully inserted into the tessellation until it meets constraints on element quality. However, the central question of any refinement algorithm is, "Where should the new points be inserted within a poor-quality element?" A reasonable answer is "as far as possible from the element vertices" [*Shewchuk* 1997]. If a new point is inserted too close to another point, the resulting small edge will create another poor tetrahedron. According to the empty sphere property of a Delaunay tessellation, the answer is the center of its circumcircle, which is as far as possible from the vertices of the element. This idea is used frequently in 2D tessellation refinement algorithms. For example, *Frey* [1987] removes poor-quality elements from a triangulation by inserting new vertices at the centers of their circumcircles while satisfying the Delaunay criterion in the triangulation. Similarly in 3D, a poor-quality tetrahedron is split into four tetrahedra by inserting a point at its *circumcenter* (the center of its circumsphere); the Delaunay property guarantees that the tetrahedron is eliminated.

Two important issues are related to the location of the new point and the updating of the tessellation:

(a) *Does location of new point preserve given boundaries and discontinuities?* An important point during construction and refinement of the tessellation is to preserve given boundaries and discontinuities. In geosciences, the boundaries are usually defined by various methods, such as a set of points with enough density obtained via the methods of surface representation, including non-uniform rational B-Splines or by a regular grid of points. Many approaches triangulate the boundaries prior to tetrahedralisation in a separate step [*Shewchuk* 1997]. To minimize the number of required points representing the boundaries, it is possible to consider the boundaries with a minimum number of points (coarse triangulation) and refine them during the refinement process. This task is completed using constraint Delaunay methods. In these methods, the refinement process is governed by rules such as *encroached* linear boundary and e*ncroached* facet [*Shewchuk* 1997; *Si* 2006]:
Encroached linear boundary: A linear boundary is *encroached* if a point (blue point in Figure 5) lies within its diametric sphere. The diametric sphere of a linear boundary is the smallest sphere containing the line. The encroached linear boundary is immediately split by inserting a point at its midpoint (red point), followed by a local updating of the tessellation. The two resulting lines have smaller diametral spheres, and may or may not be encroached themselves.
Encroached face boundary: A facet is *encroached* if a point lies within its *diametric sphere*. The *diametric sphere* of a facet is the smallest sphere that contains the facet. The encroached facet is split by inserting a point at its circumcenter, provided the new point does not encroach upon any linear boundary; otherwise, the point is not inserted and the linear boundary is encroached upon its split (Figure 6).

There are other approaches in which the refinement process may be constrained, such as the boundary elements (facets) must remain unchanged

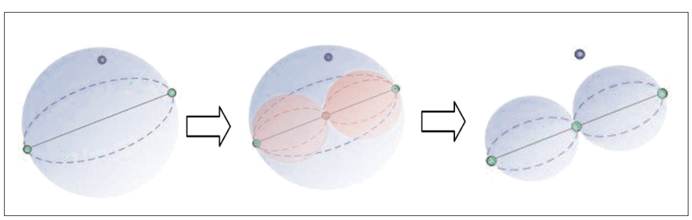


Figure 5: Any encroached linear boundary is split by inserting a point at its midpoint.

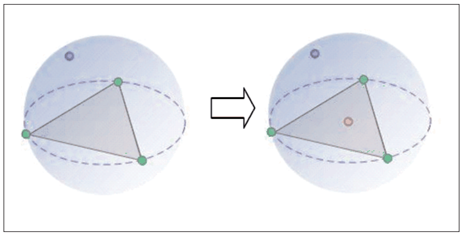


Figure 6: Any encroached facet boundary is split by inserting a point at its circumcenter.

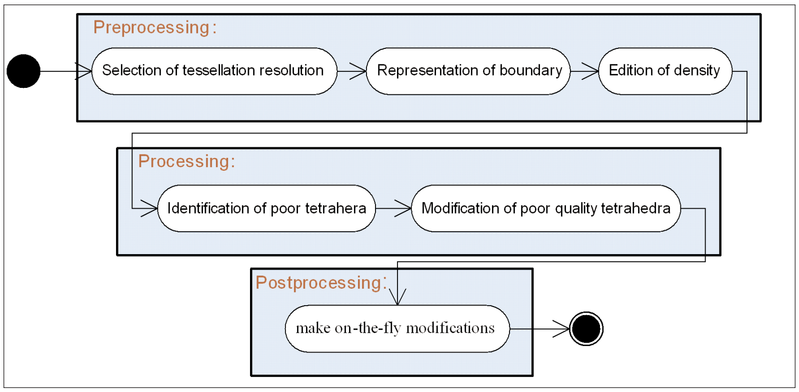


Figure 7: The steps of the proposed adaptive tessellation refinement.

during the tetrahedralization, that result in a non-Delaunay tessellation.

(b) *How is the tessellation updated?* The existing refinement methods are either static or dynamic. In "static" refinement methods, when an additional point is inserted into the tessellation, the tetrahedron containing the point is divided into four tetrahedra, each having the new point as their vertex, and no local Delaunay updating is allowed. Therefore, static refinement methods produce a non-Delaunay tessellation. An example of the static methods is the method used in LaGriT. This problem is solved in "dynamic" methods where the tessellation is locally updated after each point insertion using dynamic operations called "flips" [*Hashemi et al.* 2008]. After each point insertion, flips operations change the tetrahedra configuration in order to guarantee the Delaunay empty-circumsphere criterion [*Joe* 1991; *Shewchuk* 2005].

In the following section, we present a new dynamic refinement method for the tessellations used in geoscientific application.

4. Proposed Refining Algorithm for 3D Delaunay Tessellation

Our method for generation and refinement of an adaptive tessellation begins with the proposed tessellation, which is conformed to represent the complexity of phenomena, considering the discontinuities and the shape and size criteria. The method consists of three stages, namely refinement preprocessing, processing, and post-processing (Figure 7).

4.1 Refinement Preprocessing

In the preprocessing stage, a desired resolution for the tessellation is selected, the boundaries are defined, and the density of the data is edited. These actions are described in more detail in the following sub-sections:

a) Resolution selection*:* The tessellation resolution is a criterion used to reach a given accuracy for modeling. It depends on many factors, such as the physical nature of the phenomena being modeled and the geometrical complexity of the spatial framework [*Bower et al.* 2005]. For example, a tessellation with high resolution is usually required in regions with complex geometry or with a significant change of properties (such as hydraulic head, solute concentration, or temperature). A tessellation with lower resolution is sufficient for smooth regions. Using these criteria, the minimum and maximum length for the tetrahedra edge (L_{min}, L_{max}) is defined. In addition, the minimum and maximum variation of the field properties along the tetrahedra edge (Δf_{min}, Δf_{max}) is selected.

b) Boundaries representation: As mentioned, it is important to represent an object, or phenomenon discontinuities and boundaries, with its surrounding environment. For example, for a hydrogeological system modeling, we need to consider the faults, folds, and other objects in a given region. As another example, in a marine environment, we need to consider the air-water boundary layer, coasts, islands, and other oceanographic boundaries. The boundaries are important features that separate regions with different properties. In our method, the discontinuities and boundaries of a phenomenon are represented by considering a set of points on each side of them. The density of the points representing the boundaries has to be sufficient to ensure that tetrahedra will not extend beyond the boundaries or discontinuities. In this method, the points density for the boundaries is selected as the length between two neighbor boundary points, equal to $L_{min}/2$. Then, the boundary facets remain unchanged during the tetrahedralisation (Figure 8).

c) Density edition: As previously mentioned, due to the measurement techniques in geosciences, geoscientific data usually is composed of series of unconnected points with very irregular distribution.

For example, in hydrological applications, the data collected in a borehole are generally closely spaced vertically, but very sparse horizontally. Similar distribution is observed for marine data where instruments such as CDT (Connectivity-Temperature-Depth) are designed for vertical data recording.

In the preprocessing step, the density of the point set is edited, based on the suitable shortest interval in the tessellation. This task can be completed with a dynamic "deletion" operation. For this purpose, our algorithm considers a list of all tetrahedra present in the existing tessellation and each tetrahedron is tested. If there is at least one edge with a length smaller than the minimum length ($L < L_{min}$), one of the vertices of the edge, which is selected based on the nearest neighbor analysis and a defined tolerance, is deleted. Hence, the point with the smaller nearest neighbor distance is deleted (Figure 9).

To remove points from the initial tessellation, deletion consists of locating the point to be deleted from the tessellation and reconfiguring the tetrahedra incident to the point with a sequence of *flips* operations. Each modified tetrahedron following the deletion operation must be inserted or updated into the dynamic list to density test.

4.2 Refinement Processing

The processing stage identifies the poor-quality tetrahedra and refines them as discussed in the following subsections.

a) Poor-quality tetrahedra identification: To identify poor-quality tetrahedra in geosciences, we propose to define the criteria which specifically address both spatial and physical aspects of the problem. The geometric criteria are related to the *shape and size* of elements, while the physical criteria are based on a *physical variable criterion.* Some examples of geometric and physical criteria are:

- *Edge criterion*: the tetrahedra edge length must be less than a predefined tolerance for maximum length (L_{max}) and greater than a predefined tolerance for minimum length (L_{min}).
- *Aspect-ratio criterion:* let r and r' be the circumsphere and the inscribed sphere radiuses of of the tetrahedra must be greater than a user- a tetrahedron. The ratio of these values $\left(\beta = \frac{r}{r'}\right)$ supplied tolerance (β_{min}).
- *Volume criterion:* the volume of the element must not be greater or less than V_{max} or V_{min}; the condition $V < V_{min}$ is utilized to identify flat tetrahedra.

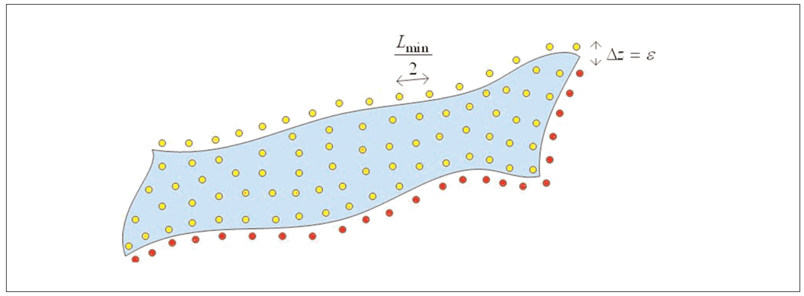


Figure 8: A boundary is represented by generation of two sets of points on both sides (red and yellow points) with a very small distance between them ($\Delta z = \varepsilon$).

- *Physical field variable criterion:* The gradient of the properties values along any edge of the tetrahedron must be less than a predefined tolerance for maximum gradient (Δf_{max}). Therefore, the tetrahedra which do not satisfy these constraints are identified as poor quality tetrahedra.

b) Poor-quality tetrahedra handling: In our method, since the boundaries are defined with a high density (the length between two neighbor boundary points is equal to the shortest edge in the tessellation) in the preprocessing step, they remain unchanged during the refinement process. Therefore, the tessellation is refined by adding points within all poor-quality elements using the "insert" operation (Figure 10). For tessellation refinement, each tetrahedron is tested separately for refinement. This test is done based on the defined geometric and physical criteria. Thus, a poor-quality tetrahedron is refined by inserting an additional point in the center of its circumsphere. Following a point insertion, the configuration of the adjacent tetrahedra is changed by flips to

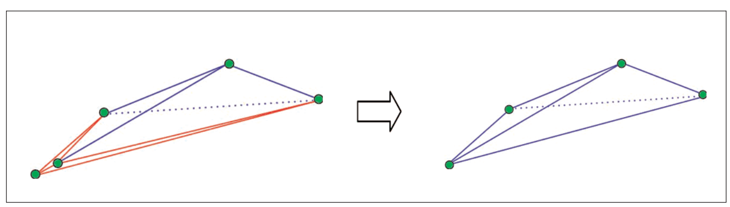


Figure 9: Tessellation refinement based on a tessellation element edge length criterion.

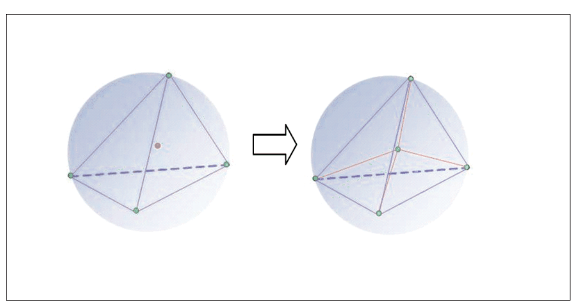


Figure 10: A poor-quality tetrahedron is split into four tetrahedra by inserting a point at its circumcenter.

satisfy the empty sphere criterion of DT. Therefore, each modified tetrahedron following the optimization operation must be tested for its quality. This process is repeated until all tetrahedra respect the mentioned conditions.

For each newly-created point, we need to determine its physical attributes. For this purpose, different interpolation methods, including the nearest neighbor method, inverse-distance weighted averaging (IDWA) method, kriging method, and the natural neighbor interpolation method can be used. 3D Voronoi diagram, which is the dual graph of the Delaunay tetrahedralisation, can not only be used to clearly define adjacency relations for each point in the 3D space, but can also be used as a basis for spatial interpolation [*Ledoux and Gold* 2004] among a given set of points. In natural neighbor interpolation methods, the neighbors of a point *p* are the Voronoi polytops that share a common face with the polytop of the point *p*. In order to proceed with the interpolation at a location *p*, first the point *p* is inserted in the tessellation, and then the volumes of the neighboring polytops are computed. Next, the point *p* is removed from the tessellation and the volumes of the polytops are re-computed. The weight w_j of each neighbor corresponds to the ratio of the difference of these volumes for each neighbor and the volume of the polytop of the point *p*. Therefore, the interpolated value at the location *p*, z_p is computed by the following equation:

$z_p = \sum_j^n w_j z_j$, where z_j *is* the attribute of each data point.

4.3 Refinement Post-processing

In the post-processing stage, the user can modify and adjust the density of the tessellation element locally, since the data structure of the tessellation is dynamic. It is possible to make on-the-fly modifications in the tessellation and explore it interactively for the extraction of information. For example, we can remove or insert faults or modify the location of wells in a hydrogeological system without having to rebuild the whole tessellation. This will help users to perform different "what if?" scenarios during the exploration process for better understanding and analysis of a complex geoscientific phenomenon. It is also possible to analyze and make some spatial queries, such as k-neighbour queries, on tessellation elements and to modify the tessellation over time if required. Figure 11 presents different steps of the proposed refinement algorithm.

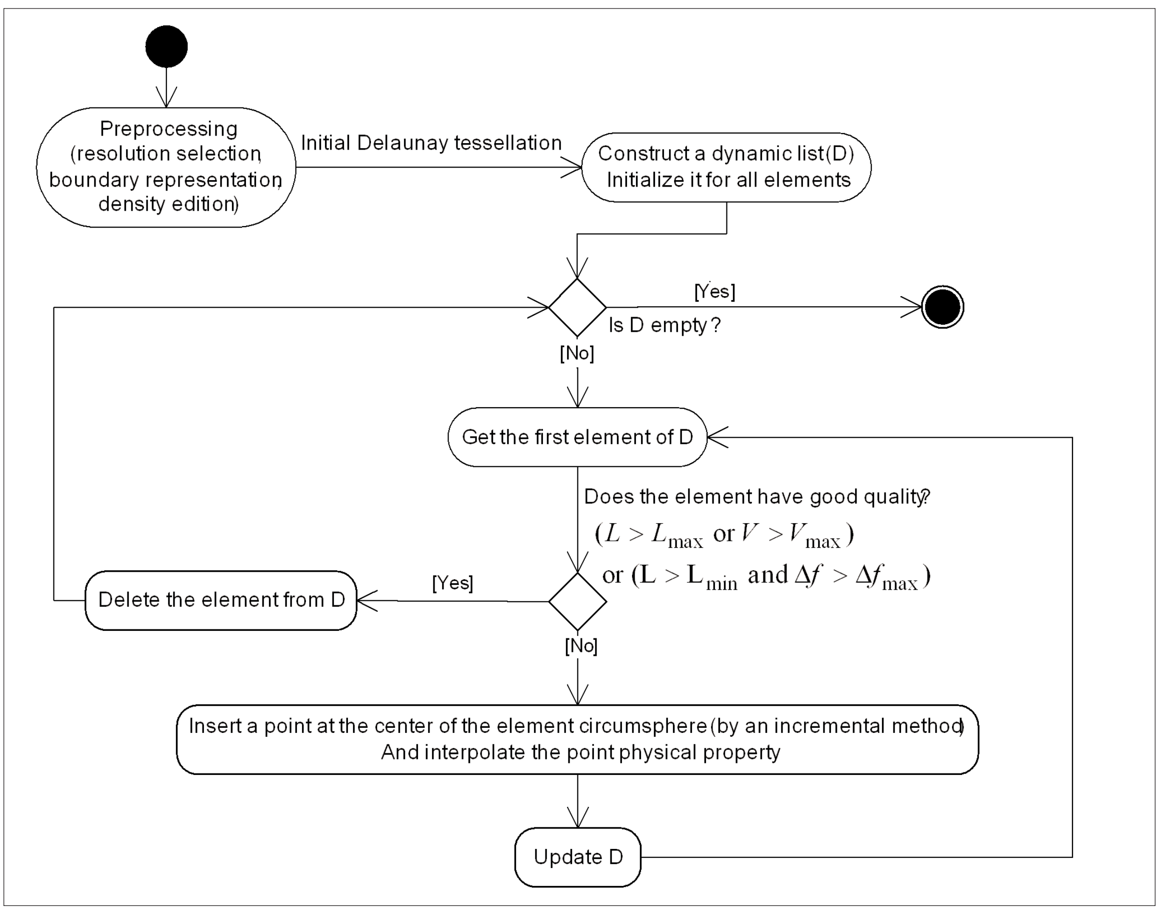


Figure 11: Flowchart of the algorithm for an adaptive refinement of a Delaunay tessellation.

5. Results and Discussion

The proposed algorithm for an adaptive refinement of a Delaunay tessellation was implemented in Borland Delphi programming language and runs under Microsoft Windows XP. For evaluation purposes, we applied the algorithm to a set of 1 292 points that were collected in four boreholes of a hydrogeological system in the Olkiluoto site[3] [*Ahokas and Koskinen* 2005]. Olkilutoto Island is located on the west coast of Finland. The bedrock in the site is mainly characterized by a low permeability gneiss rock intersected by several fracture zones. The covered area was about 740 m long, 1 000 m wide and about 1 000 m high (Figure 12).

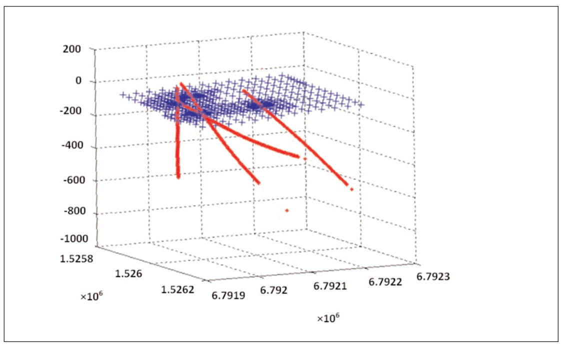
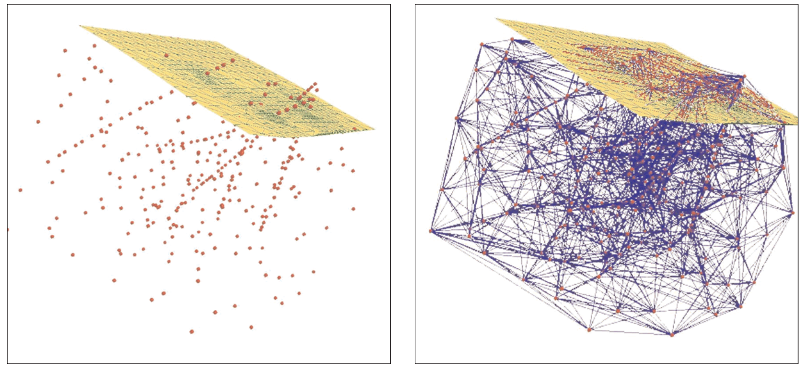


Figure 12: Data points collected in four boreholes (red points) and a planer fracture (blue points).

For the construction and refinement of a tessellation using the data set, we defined several criteria for the size and physical properties of the tessellation elements. These criteria were selected based on the experts' knowledge of the field for this specific application. Based on the criteria, the minimum and maximum length for a tetrahedra edge should be 20 m and 80 m respectively, and the minimum and maximum variation of the hydraulic conductivity along a tetrahedra edge should be 10^{-12} m/s and 10^{-8} m/s respectively.

The site is characterized by the presence of a discontinuity in the region, which is a fault. The fault is modeled using a set of 465 points with a resolution of 20 m (shortest length for the tetrahedra) on each side of it. The points collected from the boreholes are very dense vertically and sparse horizontally. On average, the distance between two adjacent points along a borehole axis is 2-3 m, while distance between the boreholes is 100-200 m. Therefore, using a nearest neighbor analysis and the dynamic "Deletion" operation, the density of the point set is edited based on the shortest edge criterion in the tessellation (20 m). In this step, the number of data points is decreased to 633 points.



(a) (b)

Figure 13: (a) Data set from boreholes after the refinement process, (b) The resulting tessellation on the data set.

Next, a 3D Delaunay tessellation is constructed using the data set and a number of 3 766 tetrahedra are created. When the refinement algorithm is used, about 12.1% of skinny tetrahedra and about 5.4% flat (slivers) tetrahedra were detected in the tessellation with respect to the given size and physical property thresholds (72 000 m^3). Following the application of the refinement algorithm, 153 new points are automatically inserted in the 3D tessellation and the total number of the points is increased to 786 points, resulting in a total number of 4 657 tetrahedra.

As we can see from the results, the proposed algorithm allowed us to properly detect and eliminate poorly-shaped elements from the tessellation. The proposed algorithm is not only comparable with other existing algorithms developed in the field of Computational Geometry and provides possibility of dynamic and local edition of the Delaunay tessellation, but also offers the advantage of considering physical criteria for the refinement. The mentioned algorithms are purely based on geometrical criteria and do not consider the physical criteria for the refinement process. In addition, these algorithms are, in some cases, not efficient enough to refine a tessellation with high irregular distribution of points and with a very smooth variation of physical values.

In terms of physical criteria, algorithms, such as in LaGriT, are static and are not able to satisfy the Delaunay criterion following any change in the tessellation (insertion or deletion of a point). In addition, the refinement procedure is based only on the insertion of new points in the center of each poor-quality element without considering the adjacent elements. This may result in some other poor-quality elements and the users have to rebuild the tessellation using the new data set obtained from the refinement process. Table 1 summarizes the qualitative comparison between the proposed method and the existing methods.

[3] Data from this site is used commonly by Finnish researchers and one of the authors in a research collaboration framework.

Table 1: The qualitative comparison between the proposed method and the existing methods.

Methods	Preprocessing step	Processing step			Post-processing step
		Poor-quality tetrahedra identification	Poor-quality tetrahedra handling	Interpolation	
Methods developed in computation geometry	Automatic (possible)	Based on pure geometric criteria	Local updating (dynamic (incremental) methods)	No	Yes
Methods developed for geosciences (such as LaGriT)	Manual	Based on both geometric and physical criteria	Global updating (static methods)	Yes	No
Proposed method	Automatic	Based on both geometric and physical criteria	Local updating (dynamic (incremental) methods)	Yes	Yes

In order to have a more practical comparison, we have compared our results with the ones obtained from LaGriT in terms of the number of poor-quality elements and the resulted tessellation elements after the refinement process. Here we should mention that since the preprocessing of points in LaGriT is manual, we have not used that procedure. Instead, the data set that resulted from our preprocessing stage was used for refinement in LaGriT. Using the same geometrical and physical criteria, the same number of poor-quality elements was detected (12.1% and 5.4% skinny and flat tetrahedra). The total number of the tetrahedra created using the software was 4 827 following the refinement of the tessellation. However, the resulted tessellation from LaGriT respects neither the Delaunay criterion nor the minimum geometric and physical thresholds (minimum length or physical variation of the property). Therefore, our method presents a great advantage over the refinement method employed by LaGriT for geosciences applications. This is because, in most of the geoscientific applications, the estimation of the given physical properties is usually based on the proximity criterion and is obtained by an interpolation operation. The Delaunay criterion ensures that we have an optimal tessellation of the space, meaning that the adjacency between the points is more adequately defined, which better respects the proximity criterion.

5.1 Efficiency Analysis of the Proposed Algorithm

The time complexity of the proposed dynamic Delaunay refinement algorithm is determined as follows:

Refinement preprocessing: The 3D DT of a set of n points in R^3 is constructed using the incremental algorithm. The performance of this operation is $O(n^2)$ in the worst case. In the case of point deletion, the time complexity is $O(es)$ where e is the number of *ears* of the point star and s is the point degree. The number of tests to determine poor-quality tetrahedra corresponds to the number of tetrahedra in the structure.

Refinement processing: The processing stage includes updating the data structure by point insertion in the poor-quality tetrahedra. In the case of point insertion, if the new point conflicts with k simplices, then $O(k)$ time is needed to insert it and compute its attribute based on natural neighbor interpolation. For any insert or delete operation, the dynamic list should be updated, which takes $O(\log n)$.

Therefore, in the worst case scenario, where there are both very large and very small tetrahedra, $O((m + l)^2)$time is needed, where m is the size of the final tessellation and l is the number of deleted points. However, in practice, the algorithm will most likely be faster.

6. Conclusions

In this paper we have discussed the construction of a 3D adaptive tessellation for 3D spatial modeling in geosciences. Despite the numerous advantages of 3D VD and DT for 3D modeling and representation, they have some limitations when it comes to representing more complex geoscientific phenomena where the representation is based on datasets which are highly irregular and non-homogeneously distributed (e.g. data obtained from boreholes). In these cases, DT may result in very poor-quality mesh elements. These poor-quality elements, such as flat and skinny tetrahedra, are not suitable for 3D spatial modeling purposes. Hence, in this paper we have developed a 3D refinement method based on the dynamic Delaunay tetrahedralisation which deals with those elements and creates a tessellation that can more accurately represent complex geoscientific phenomena. The proposed algorithm, through its dynamic operations such as insert, delete, and flips, offers an efficient way to refine poor-quality elements in 3D- DT-based tessellations. The proposed method has several advantages over the existing methods and provides an automatic and on-the-fly refinement capability by respecting more accurately the proximity criterion and by creating an optimal tessellation for the representation of complex spatial phenomena in geosciences.

Acknowledgement

The authors would like to acknowledge the support of the GEOIDE Network under GeoTopo3D project, entitled: "Development of 3D Predictive Modeling Platform for Exploration, Assessment and Efficient Management of Mineral Petroleum and Groundwater Resources." The authors would also like to acknowledge Dr. Christopher Gold from the University of Glamorgen, and Dr. Hugo Ledoux from Delft University of Technology for providing us with the source code for the construction of 3D DT and VD.

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